Necessary and sufficient condition for vanishing super discord

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We investigate the super quantum discord based on weak measurement, which is an extension of quantum discord defined for projective measurement. We provide a few equivalent conditions on zero super quantum discord by using quantum discord, classical correlation and mutual information. As an application, we investigate the super discord in optimal case of the protocol of state discrimination assisted by an auxiliary system. It is shown that the super discord always exists while one side quantum discord and entanglement is not presented in the optimal case. This result expands the present knowledge of quantum correlation as a resource in quantum information processing and super discord can be regarded as a candidate for correlation needed in assisted state discrimination scheme.

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I. INTRODUCTION

Quantum measurement plays a key role in quantum mechanics. It has some interesting quantum properties which are different from our everyday lives including the collapsing of wavefunction, compatible observables and the contextuality phenomena. For a quantum measurement, one need to construct a set of orthogonal projection operators which is corresponding to the observable eigenvector spaces of a Hermitian operator. The possible outcomes of the measurement correspond to the eigenvalues of the Hermitian operator. This is the standard von Neumann measurement or projective measurement [1]. Recently, the formalism was generalized to the POVM measurement which stands for 'Positive Operator Valued Measure' [2], and can really capture many phenomena such as additional randomness measurement outcomes and many others beyond projective measurement.

However, the measurement of quantum state inevitably disturbs the quantum system which in turn determines our retrieved knowledge about the measured system. In order to make the least influence on original quantum state, or the extent of the influence is variational by active control or depending on environment, one may introduce a measurement that induces a partial collapse of a quantum state. This is the so-called "weak measurement" [3–5]. An interesting thing is that quantum states can be retrieved with a nonzero success probability when the interaction between the system and measurement apparatus is weak [6]. It has been shown that any generalized measurements can be decomposed into a sequence of weak measurements and therefore weak measurement are universal [8]. The reversing process has been attracting much attention theoretically and also realized experimentally [9, 10] due to its potential applications in quantum information processing recently [7].

Searching for quantum correlation in composite system and identifying their role in quantum information processing is one of the fundamental problems in quantum mechanics. Quantum entanglement is extensively regarded as a crucial role in quantum teleportation and super dense coding, etc [2]. Quantum discord [11–13], which is beyond quantum entanglement, can effectively grasp the role of quantumness of correlations and is different from the classical correlation. Quantum discord is shown to be present in deterministic quantum computation with one qubit (DQC1) [14], as a resource in remote state preparation [15], and the consumed discord bounds the quantum advantage in encode information [16]. Quantum dissonance (or one side discord) is shown to be required in optimal assisted discrimination [18–20]. A fundamental question we need to solve is that how the correlation exists in composite quantum system. For entanglement, there are many criteria constructed to detect entangled states [17, 21, 31]. For the quantum discord, the vanishing of quantum correlation means the quantum state must be the form of so called classical-classical (CC) state, classical-quantum (CQ) state or quantum-classical (QC) state [22, 23, 26].

Very recently, Singh and Pati introduced the concept of super quantum discord induced by weak measurement [27]. They argued that super quantum discord can capture much more quantum correlation in the sense that the super quantum discord is always larger than the normal discord induced by the strong (projective) measurement. Furthermore, super discord can result in an improvement on the entropic uncertainty relations [24, 25]. Now we ask, what is the criterion by which super quantum discord exists in quantum system? Can super discord exist in some quantum information model where quantum discord and entanglement doesn't exist? In this paper, we provide a necessary and sufficient condition for super discord in terms of classical correlation, mutual information, and normal discord. We further illustrate that super discord can present in optimal assisted state discrimination on both sides, where only one side of quantum discord is present and entanglement is totally not needed.

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This paper is organized as follows. In Sec. II, we recall some definition and property of super discord. In Sec. III, we provide a series of necessary and sufficient condition on vanishing super discord. An illustration of super discord present on both sides in optimal assisted state discrimination is given in In Sec. IV. Finally we summary in Sec. V.

II. THE CONCEPT AND PROPERTY OF SUPER DISCORD

Consider the bipartite state ρ on the space $\mathcal{H}_A \otimes \mathcal{H}_B$. Let $\{\pi_k\}$ be one-dimensional von Neumann projectors, and the probability $p_k = \mathrm{Tr}(I \otimes \pi_k)\rho(I \otimes \pi_k)$. The completeness of the operators $\{\pi_k\}$ implies the formula $\sum_k p_k = 1$. Next, $S(\rho) := -\mathrm{Tr}\,\rho\log\rho$ is the von Neumann entropy and "log" denotes "$\log_2$" throughout the paper. We refer to ρ_A, ρ_B as the reduced density operators of ρ. Then we denote $I(\rho) := S(\rho_A) + S(\rho_B) - S(\rho)$ as the mutual quantum information and $C(\rho) := \max_{\pi_k} I\Big(\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k)\Big)$ as the classical correlation [11, 12, 28]. Both of them are non-negative because the mutual information is non-negative [2].

The quantum discord for ρ is defined as the difference between the mutual information and classical correlation [12, 22]

$$
\begin{aligned}
D(\rho) &= I(\rho) - C(\rho) \\
&= S(\rho_B) - S(\rho) + \min_{\pi_k} \sum_k p_k S\Big(\frac{(I \otimes \pi_k)\rho(I \otimes \pi_k)}{p_k}\Big) \quad (1)
\end{aligned}
$$

It is known that [23, 26] the ("right") discord is zero if and only if $\rho = \sum_i p_i \rho_i \otimes |\varphi_i\rangle\langle\varphi_i|$, where the $|\varphi_i\rangle$ are o. n. basis. This is the so-called classical state in the system B.

Next we recall the super quantum discord $D_w(\rho)$ for two-qubit states ρ introduced in [27, Eq. (9)]. It is defined as

$$
D_w(\rho) := \min_{\{\pi_0,\pi_1\}} S_w(A|\{P^B(x)\}) - S(A|B), \qquad (2)
$$

where the conditional entropy $S(A|B) = S(\rho) - S(\rho_B)$ and

$$
S_w(A|\{P^B(x)\}) = p(x)S(\rho_{A|P^B(x)}) + p(-x)S(\rho_{A|P^B(-x)}), \qquad (3)
$$

$$
p(\pm x) = \mathrm{Tr}\Big((I \otimes P^B(\pm x))\rho(I \otimes P^B(\pm x))\Big), \qquad (4)
$$

$$
\rho_{A|P^B(\pm x)} = \frac{1}{p(\pm x)} \mathrm{Tr}_B \Big((I \otimes P^B(\pm x))\rho(I \otimes P^B(\pm x))\Big), \qquad (5)
$$

$$
P(x) = \sqrt{\frac{1-\tanh x}{2}}\pi_0 + \sqrt{\frac{1+\tanh x}{2}}\pi_1, \qquad (6)
$$

$$
P(-x) = \sqrt{\frac{1+\tanh x}{2}}\pi_0 + \sqrt{\frac{1-\tanh x}{2}}\pi_1, \qquad (7)
$$

and $x \in R \setminus \{0\}$ is a parameter describing the strength of measurement process. By Eq. (2), we have $D_w(U \otimes V\rho U^\dagger \otimes V^\dagger) \leq D_w(\rho)$ with arbitrary unitary U, V. One may similarly obtain $D_w(U \otimes V\rho U^\dagger \otimes V^\dagger) \geq D_w(\rho)$, so we have

$$
D_w(U \otimes V\rho U^\dagger \otimes V^\dagger) = D_w(\rho). \qquad (8)
$$

That is, the super discord is invariant up to the local unitary. This property is the same as that of normal discord.

By Eqs. (6) and (7), we obtain the completeness relation

$$
\pi_0 + \pi_1 = P(x)^\dagger P(x) + P(-x)^\dagger P(-x) = I. \qquad (9)
$$

By Eqs. (4) and (9), we see that the probability sum is equal to one:

$$
p(x) + p(-x) = 1. \qquad (10)
$$

Using the concavity of von Neumann entropy and Eqs. (3) and (5), we easily obtain $I(\rho) \geq D_w(\rho)$. By combining the Theorem of [27], we have

$$
I(\rho) \geq D_w(\rho) \geq D(\rho) \qquad (11)
$$

for any two-qubit states. However these three quantities are not quantitatively related to the classical correlation. Indeed, it follows from [29, Fig.1] that the difference $C(\rho) - D(\rho)$ can be either positive or negative for two-qubit Bell diagonal states ρ in [29, Eq. 18], see also [30, Eq. 8]. Nevertheless, we will determine the relations between classical correlation, mutual information, super discord and discord for the product states in next section.

III. CONDITION FOR ZERO SUPER DISCORD

Similar to the case of discord we ask the following question: what are the states ρ whose super discord is zero? By Eq. (11) and [23, 26], such states ρ must be classical in the system B. However the converse is not evidently true, see Theorem 1. For this purpose we need a preliminary lemma. It is known that the classical correlation is zero for the product state [11]. We show that the inverse is also true.

Lemma 1. *Any bipartite state ρ realizing $C(\rho) = 0$ is a product state, i.e., $\rho = \rho_A \otimes \rho_B$.*

Proof. By definition, the condition $C(\rho) = 0$ implies that $I\Big(\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k)\Big) = 0$ holds for any $\{\pi_k\}$. By the subadditivity of von Neumann entropy, the state $\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k)$ is a product state. By tracing out the system A or B, we have

$$
\sum_k (I \otimes \pi_k)\rho(I \otimes \pi_k) = \rho_A \otimes \sum_k \pi_k \rho_B \pi_k \qquad (12)
$$

for any $\{\pi_k\}$. Let $\rho_B = \sum_i p_i |b_i\rangle\langle b_i|$ be the spectral decomposition, and we can assume $\rho = \sum_{ij} \rho_{ij} \otimes |b_i\rangle\langle b_j|$. By choosing $\pi_i = |b_i\rangle\langle b_i|$ in Eq. (12), we obtain $\rho_{ii} = p_i \rho_A$, $\forall i$. Using the normalization condition $\sum_i p_i = 1$ we have $\rho = \rho_A \otimes \rho_B + \sum_{i\neq j} \rho_{ij} \otimes |b_i\rangle\langle b_j|$. By replacing by this formula ρ in Eq. (12), we have

$$
\sum_k (I \otimes \pi_k)\Big(\sum_{i\neq j} \rho_{ij} \otimes |b_i\rangle\langle b_j|\Big)(I \otimes \pi_k) = 0 \qquad (13)
$$

for any $\{\pi_k\}$. Since any two summands are orthogonal, we have $(I \otimes \pi_k)\Big(\sum_{i\neq j} \rho_{ij} \otimes |b_i\rangle\langle b_j|\Big)(I \otimes \pi_k) = 0$, $\forall k$. By choosing

$$
\pi_k = \Big(\frac{1}{\sqrt{2}}|b_l\rangle + \frac{1}{\sqrt{2}}e^{i\alpha}|b_j\rangle\Big)\Big(\frac{1}{\sqrt{2}}\langle b_l| + \frac{1}{\sqrt{2}}e^{-i\alpha}\langle b_j|\Big), \qquad (14)
$$

we have $\rho_{lj}e^{i\alpha}+\rho_{jl}e^{-i\alpha}=0$ for any real α. So $\rho_{lj}=\rho_{jl}=0$ for any $j\neq l$. Thus $\rho=\rho_A\otimes\rho_B$ and the assertion follows. This completes the proof. □

Theorem 1. *The following seven statements are equivalent for the two-qubit state* ρ*:*
(a) ρ *is a product state;*
(b) ρ *has zero classical correlation;*
(c) ρ *has zero super discord;*
(d) ρ *has zero mutual information;*
(e) ρ *has equal discord and super discord;*
(f) ρ *has equal discord and mutual information;*
(g) ρ *has equal super discord and mutual information.*

Proof. (a)→(b) follows from the definition of classical correlation.

(b)→(c) By Lemma 1 we may suppose $\rho=\rho_A\otimes\rho_B$. By Eqs. (4) and (5), we obtain $\rho_{A|P^B(\pm x)}=\rho_A$. By Eqs. (3) and (10), we have $S_w(A|\{P^B(x)\})=S(\rho_A)$. Then Eq. (2) implies that $D_w(\rho)=0$, so (b)→(c) follows.

(c)→(e) follows from $D(\rho)\geq 0$ and Eq. (11).

(e)→(d). Let $\{\pi_i\}$ be the measurement basis that minimizes the super discord in Eq. (2). By [27, Eq. (11)] and Eq. (1), we have

$$D_w(\rho)\geq\sum_{k=0}^{1}p_kS\Big(\frac{(I\otimes\pi_k)\rho(I\otimes\pi_k)}{p_k}\Big)-S(A|B)\geq D(\rho).\quad(15)$$

By the hypothesis, both equalities in Eq. (15) holds. By [27, Eq. (11)] and the concavity of von Neumann entropy, the first equality holds only if $\mathrm{Tr}_B\frac{(I\otimes\pi_0)\rho(I\otimes\pi_0)}{p_0}=\mathrm{Tr}_B\frac{(I\otimes\pi_1)\rho(I\otimes\pi_1)}{p_1}$ [2]. Since the operators π_k are of rank one, the second equality implies

$$\begin{aligned}D(\rho)&=\sum_{k=0}^{1}p_kS\Big(\mathop{\mathrm{Tr}}_B\frac{(I\otimes\pi_k)\rho(I\otimes\pi_k)}{p_k}\Big)-S(A|B)\\&=S\Big(\mathop{\mathrm{Tr}}_B\frac{(I\otimes\pi_0)\rho(I\otimes\pi_0)}{p_0}\Big)-S(A|B)\\&=S\Big(\sum_{k=0}^{1}p_k\mathop{\mathrm{Tr}}_B\frac{(I\otimes\pi_k)\rho(I\otimes\pi_k)}{p_k}\Big)-S(A|B)\\&=S(\rho_A)-S(A|B)\\&=I(\rho).\end{aligned}\quad(16)$$

The second equality follows from the formula $p_0+p_1=1$, and the fourth equality from Eq. (9). It follows from Eq. (1) that Eq. (16) holds only if $C(\rho)=0$. Then Lemma 1 implies that ρ is a product state, so $(e)\to(d)$ follows.

(d)→(f). The hypothesis $I(\rho)=0$ implies that ρ is a product state. So the discord is also zero and the assertion follows.

(f)→(g). It is a corollary of Eq. (11).

(g)→(a). By Eq. (2) and the concavity of von Neumann entropy, we have

$$\begin{aligned}D_w(\rho)&=\min_{\{\pi_0,\pi_1\}}S_w(A|\{P^B(x)\})-S(A|B)\\&=S(\rho_A)-S(A|B)\\&\geq\max_{\{\pi_0,\pi_1\}}S_w(A|\{P^B(x)\})-S(A|B).\end{aligned}\quad(17)$$

So the quantity $S_w(A|\{P^B(x)\})$ is constant for any π_0,π_1. The equality in Eq. (17) holds if and only if $\rho_{A|P^B(x)}=\rho_{A|P^B(-x)}=\rho_A$ for any π_0,π_1. By Eqs. (5), (6) and (9), we have $(I\otimes\pi_k)\rho(I\otimes\pi_k)\propto\rho_A$. This fact and Eq. (1) imply that $D(\rho)=I(\rho)$. So we have proved (g)→(f)→(b)→(a), where the last relation follows from Lemma 1. This completes the proof. □

As a typical example, Theorem 1 implies that the maximally mixed state $\frac{1}{4}I\otimes I$ has zero super discord. This observation has been included as a special case of [27, Eq. 19]. Note that the equivalence in Theorem 1 does not hold for states with zero discord, because such states may be not product states.

Theorem 1 shows that super discord is ubiquitous in quantum system since that they vanish only on product state. It is widely accepted that mutual information contains both classical correlation and quantum correlation. However, they share the same vanishing condition with super discord. By Theorem 1, we can also say that super discord is larger than normal discord generally, which makes that super discord really likely reveal much more quantum correlation than quantum discord. Another superiority of super discord is that their vanishing does *not* rely on the specific side, although we have the measurement acting on "left" or "right" system. The reason is if "left" super discord is zero, then the state is a product state by Theorem 1. So the "right" super discord must be also zero. As an application, we illustrate how super discord present in optimal assisted discrimination that different from normal discord in next section.

IV. SUPER DISCORD IN OPTIMAL STATE DISCRIMINATION

We first review the scheme of state discrimination introduced by Roa, Retamal and Alid-Vaccarezza (RRA scheme) [18]. Consider two nonorthogonal states $|\psi_+\rangle$ and $|\psi_-\rangle$ is randomly prepared in one of the *priori* probabilities p_+ and p_- with $p_++p_-=1$. To discriminate the two states $|\psi_+\rangle$ or $|\psi_-\rangle$. Couple the original system to an auxiliary qubit A by a joint unitary transformation U such that

$$\begin{aligned}U|\psi_+\rangle|k\rangle_a&=\sqrt{1-|\alpha_+|^2}|+\rangle|0\rangle_a+\alpha_+|0\rangle|1\rangle_a,\\U|\psi_-\rangle|k\rangle_a&=\sqrt{1-|\alpha_-|^2}|-\rangle|0\rangle_a+\alpha_-|0\rangle|1\rangle_a,\end{aligned}\quad(18)$$

where $|k\rangle_a$ is an auxiliary state with orthonormal basis $\{|0\rangle_a,|1\rangle_a\}$, $|\pm\rangle\equiv(|0\rangle\pm|1\rangle)/\sqrt{2}$ are the orthonormal states of the system that can be discriminated.

The state of the system and ancilla qubits is now given as

$$\begin{aligned}\rho_{|\alpha_+|}=&\,p_+U\left(|\psi_+\rangle\langle\psi_+|\otimes|k\rangle_a\langle k|\right)U^\dagger\\&+p_-U\left(|\psi_-\rangle\langle\psi_-|\otimes|k\rangle_a\langle k|\right)U^\dagger.\end{aligned}\quad(19)$$

It is shown that the conclusive recognition between two nonorthogonal states relies on the existence of entanglement and discord in general case of RRA scheme [18]. However, in the optimal case or with maximum recognized probability, only one side ("right" side) discord (or dissonance) is presented [19]. An interesting question is that what kinds of

non-classical correlation can be regarded as a candidate for resource for the scheme in the optimal case? By the fact that the super quantum discord is always greater than or equal to the normal quantum discord and the equivalent condition given in Theorem 1, we guess that the super quantum discord really catch the non-classical correlation of the RRA scheme on both side.

In the following, we concentrate on the state (19) in zero "left" discord cases. Since "right" discord is always presented in the quantum system, and super discord is larger than normal discord, we have "right" super discord must be presented in the scheme. As is shown in [19], in the cases of the "left" discord disappears, the following three items must be satisfied: α is a real number, and $\alpha \geq 0$; $p_+ = p_- = \frac{1}{2}$; $|\alpha_+| = |\alpha_-| = \sqrt{|\alpha|} = \sqrt{\alpha}$ and the case is indeed the optimal assisted state discrimination. For convenience, we set $\alpha_+ = c$ be a real number. Then the state ρ in (19) reduced to

$$\rho_c = \frac{1-c^2}{2}(I \otimes |0\rangle\langle 0|) + |0\rangle\langle 0| \otimes \\ [c^2|1\rangle\langle 1| + \frac{\sqrt{2}c\sqrt{1-c^2}}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|)]. \quad (20)$$

We use the weak measurement $P(x) \otimes I, P(-x) \otimes I$ to act the state ρ_c in (20), where $P(x), P(-x)$ is given by Eqs. (6) and (7), and $\pi_0 = |\psi\rangle\langle\psi|, \pi_1 = |\widetilde{\psi}\rangle\langle\widetilde{\psi}|$, $|\psi\rangle = \cos\theta|0\rangle + e^{i\varphi}\sin\theta|1\rangle$, $|\widetilde{\psi}\rangle = \sin\theta|0\rangle - e^{i\varphi}\cos\theta|1\rangle$. Then the weak "left" conditional entropy for this state is given by

$$S_w(B|\{P^A(x)\}) = -p(x)[\lambda_+(x)\log\lambda_+(x) \\ +\lambda_-(x)\log\lambda_-(x)] - p(-x)[\lambda_+(-x)\log\lambda_+(-x) \\ +\lambda_-(-x)\log\lambda_-(-x)], \quad (21)$$

here $p(x) = \frac{1}{2}(1 - \tanh(x)\cos(2\theta)c^2)$, and,

$$\lambda_\pm(x) = \frac{1}{2(1-\tanh(x)\cos(2\theta)c^2)}(1 - \tanh(x)\cos(2\theta)c^2 \\ \pm(1 - 2c^2 + 2c^4 - 2c^2\tanh(x)\cos(2\theta) \\ +(2c^2 - c^4)(\tanh(x)\cos(2\theta))^2)^{\frac{1}{2}}), \quad (22)$$

and $\lambda_\pm(-x)$ can be similarly defined. After calculation, we find that $S(AB) = S(A)$. Let $D_w(B:A) = D_w(\rho)$ in Eq. (2) by exchanging systems A and B. From Eqs. (2) and (21) we have

$$D_w(B:A) = \min_{\{\pi_i^A\}} S_w(B|\{P^A(x)\}) = \min_\theta S_w(B|\{P^A(x)\}), (23)$$

and $D_w(B:A)$ is a function of x and c. In Fig. 1, we have plotted the picture of $D_w(B:A)$. We can see that for all $0 < c < 1$, the super discord increases with the decreasing of the strength of the measurement x. When $x \to +\infty$, the weak measurement reduces to the strong measurement and the super discord approach to normal discord. An interesting fact is that the normal discord and entanglement are always zero in this optimal case. Thus, we have shown that super discord can be regarded as a resource in optimal assisted state discrimination.

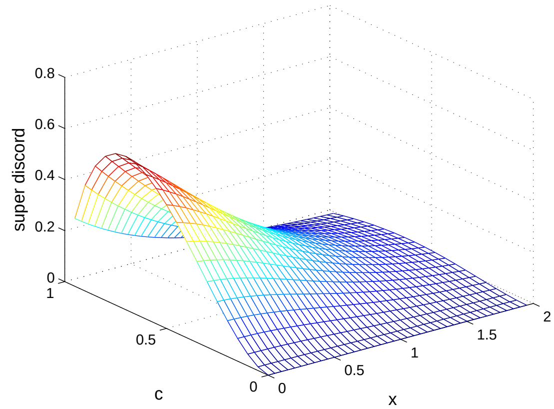

FIG. 1. super discord in the optimal case of assisted state discrimination as a function of $\alpha_+ = c$, and the strength x in the measurement process for $0 \leq \alpha_+ \leq 1$ and $0 \leq x \leq 2$.

V. SUMMARY AND DISCUSSION

In this paper, we have characterized the necessary and sufficient condition for zero super discord. It is shown that the vanishing of super discord is equivalent to vanishing of classical correlation, mutual information *etc.* So super quantum discord is a kind of quantum correlation which ubiquitously exists in quantum system. Further more, super quantum discord can be present in some quantum information processing task where entanglement is totally not necessary and only one side quantum discord is presented. An interesting question is to generalize the results of this paper to higher dimensions.

One fundamental problem in quantum information is to quantify correlations. Quantum discord emerges in separating total correlations into quantum and classical parts. Some evidences show that all correlations behave as if they were exclusively quantum [32]. In this paper, we confirm this concept by showing that super quantum discord vanishes only when mutual (total) information vanishes. This extends the regime of quantumness of correlations to all bipartite quantum states except the product state.

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